T-TEST

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Sample‘t’ statistics take up the testing of hypothesis about the population mean with smaller sample sizes (sample size ≤ 30), when the population standard deviation is unknown. Types of Sample‘t’ statistics are: 1)One-sample t-test
2)Two-sample t-test or Independent Samples t-Tests
3)Paired t-test

SAMPLE T-TEST

Introduction

Sample'**t**' statistics take up the testing of hypothesis about the population mean with smaller sample sizes (sample size ≤ 30), when the population standard deviation is unknown. Types of Sample't' statistics are:

i) One-sample *t*-test
ii) Two-sample *t*-test or Independent Samples t-Tests
iii) Paired *t*-test

i) One-Sample t-Test

One sample t-test is used to determine if the mean of a sample is different from a particular value for a single population.

Case Analysis-1

Problem

A consultancy firm claims that the average salary offered by its firm to the MBA fresher's is Rs. 22,000 per month. A sample of 15 management students of a college was taken and the information was collected about their starting salary, to test the claim of the firm.

Table-1: Sample Data

Student Number	1	2	3	4	5	6	7	8
Salary in 000's of rupees	12,000	11,000	23,000	18,000	25,000	19,000	17,000	19,000

Student Number	9	10	11	12	13	14	15
Salary in 000's of rupees	22,000	17,000	21,000	24,000	28,000	21,000	17,000

The hypotheses for the analysis are:

Null hypothesis-H_0: The average salary of MBA fresher's is Rs.22, 000. (μ= 22,000)

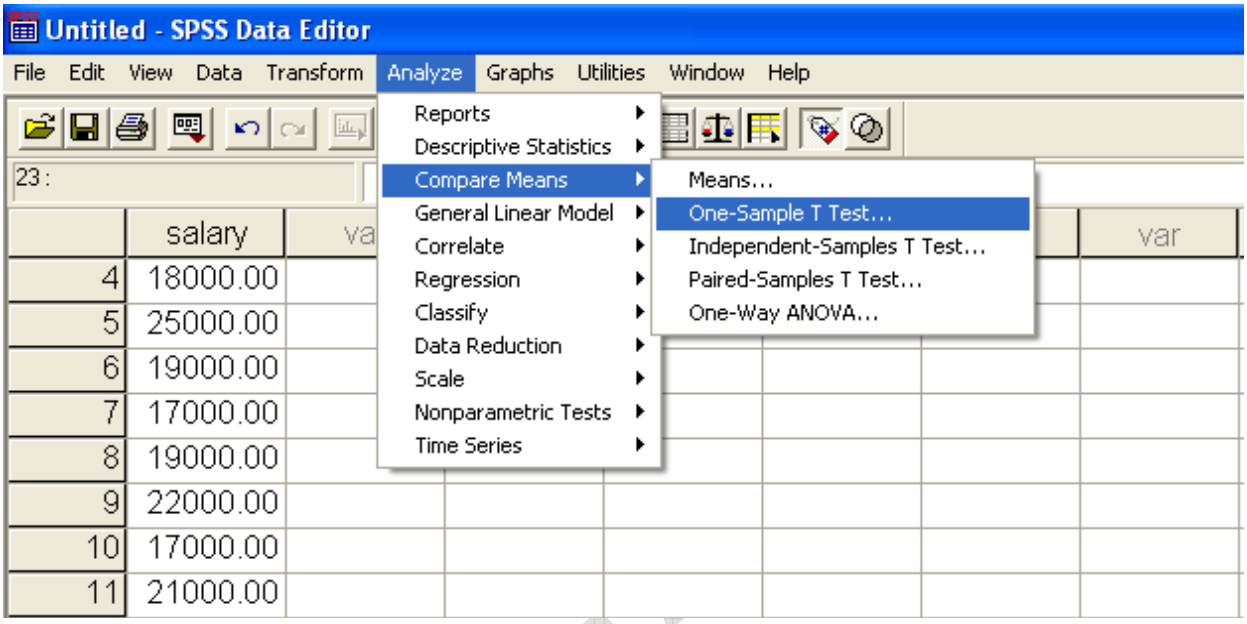
Alternative Hypothesis- H_1: The average salary of MBA fresher's is not Rs.22, 000 ($\mu \neq$ 22,000) (two-tailed test)

Input Data

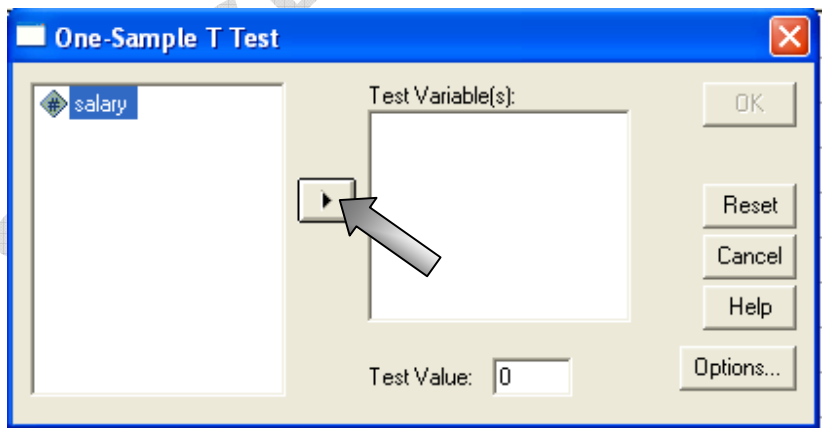
The salaries of 15 management students are treated as input data for the analysis.

PERFORMING THE ANALYSIS WITH SPSS

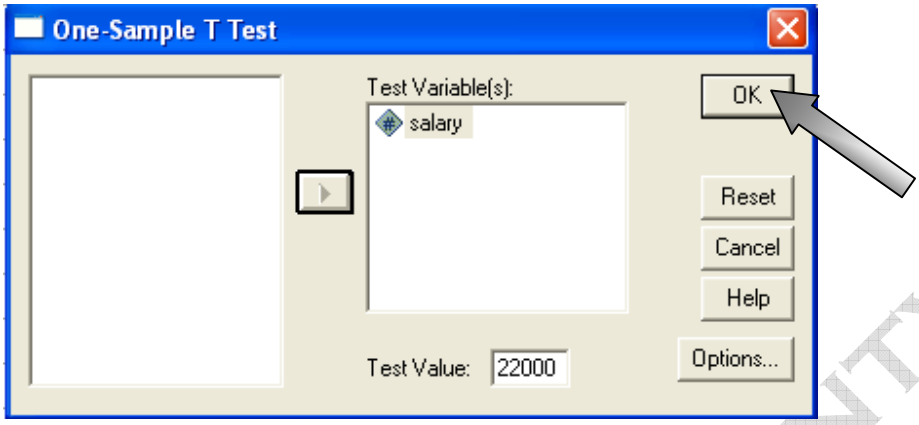
FOR SPSS VERSION 11, CLICK ON ANALYZE COMPARE MEANS ONE-SAMPLE T TEST. THIS WILL BRING UP THE SPSS SCREEN DIALOGUE BOX AS SHOWN BELOW.



AFTER CLICKING ONE SAMPLE T TEST, THIS WILL BRING UP THE FOLLOWING SPSS SCREEN DIALOGUE BOX.



SELECT THE DEPENDENT VARIABLE AND CLICK IT TO MOVE IT TO TEST VARIABLES BOX. IN THIS CASE, WE ARE COMPARING IF THE AVERAGE SALARY IS RS.22, 000 OR NOT. SO WE SHOULD ENTER 22,000 INTO THE TEST VALUE BOX.



FINALLY CLICK OK OF THE MAIN DIALOGUE BOX.

SPSS OUTPUT

THE SPSS OUTPUTS OF THE ANALYSIS ARE GIVEN IN FOLLOWING TABLES.

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SALARY	15	19600.00	4595.02837	1186.431

THE TABLE TELLS THAT THE AVERAGE SALARY OF 15 MBA FRESHER'S IS Rs19, 600 WITH STANDARD DEVIATION 4595.02837.

One-Sample Test

	Test Value = 22000					
					95% Confidence Interval of the Difference	
	t	df	Sig. (2-tailed)	Mean Difference	Lower	Upper
SALARY	-2.023	14	.063	-2400.0000	-4944.64	144.6419

FROM THE OUTPUT, T = -2.203 WITH 14 DEGREES OF FREEDOM.

DECISION

REJECT THE NULL HYPOTHESIS IF P-VALUE (SIG. (2-TAILED)) ≤ 0.05

INTERPRETATION

THE P-VALUE IS 0.063 AND IT IS MORE THAN 0.05 (5% LEVEL OF SIGNIFICANCE), SO WE ACCEPT THE NULL HYPOTHESIS AND REJECT THE ALTERNATIVE HYPOTHESIS AT 5% LEVEL OF SIGNIFICANCE. IT IS CONCLUDED THAT THE AVERAGE SALARY OF MBA FRESHER'S OFFERED BY THE CONSULTANCY FIRM IS NOT Rs. 22,000.

CASE ANALYSIS-1

PROBLEM

THE AVERAGE SALE OF A CHOCOLATE BAR OF BRAND X IS 130 DOZENS PER MONTH. AFTER AN ADVERTISEMENT CAMPAIGN THE SALE IS CLAIMED TO INCREASE. A SAMPLE OF 20 STORES IN A LOCALITY IS CHOSEN TO TEST THE VALIDITY OF THE CLAIM. THE SALE OF CHOCOLATE BARS PER STORE IS GIVEN BELOW.

TABLE-1: SAMPLE DATA

STORE NUMBER	1	2	3	4	5	6	7	8	9	10	11
SALES IN DOZENS	132	122	152	136	142	120	118	119	125	156	89

STORE NUMBER	12	13	14	15	16	17	18	19	20	21	22
SALES IN DOZENS	98	100	125	102	136	135	126	145	148	156	165

NULL HYPOTHESIS-H_0: THE AVERAGE SALE OF CHOCOLATE BARS PER STORE IS 130 DOZENS (μ= 130 DOZENS).

ALTERNATIVE HYPOTHESIS- H_1: THE SALE AVERAGE SALE OF CHOCOLATE BARS PER STORE IS MORE THAN 130 DOZENS (μ >130 DOZENS). **(ONE-TAILED TEST)**

SPSS OUTPUT

One-Sample Statistics

	N	Mean	Std. Deviation	Std. Error Mean
SALES	22	129.4091	20.39061	4.34729

One-Sample Test

	Test Value = 130					
	t	df	Sig. (2-tailed)	Mean Difference	95% Confidence Interval of the Difference	
					Lower	Upper
SALES	-.136	21	.893	-.5909	-9.6316	8.4498

From the output, t = -.136 with 21 degrees of freedom.

Decision

Reject the null hypothesis if p-value (Sig. (2-tailed)) ≤ 0.05

The p-value for one-tailed test would be $\frac{\text{Sig.}(2-\text{tailed})}{2}$

Interpretation

The p-value is $\frac{\text{Sig.}(2-\text{tailed})}{2} = \frac{0.893}{2} = 0.446$ **and it is less than 0.05 (5% level of significance), so we reject the null hypothesis and accept the alternative hypothesis at 5% level of significance. Therefore it is concluded that the mean sales of chocolate bar has increased due to the advertisement campaign.**

SPSS Command

1. **Click on ANALYZE at the SPSS menu bar (in older versions of SPSS, click on STATISTICS instead of ANALYZE).**
2. **Click on COMPARE MEANS followed by ONE SAMPLE T TEST.**
3. **Select the test variable and move it TEST VARIABLE(S) box. Fill the box TEST VALUE with appropriate hypothesized value and click CONTINUE.**
4. **Select OK of the main dialogue box.**

ii) Two-sample t-test or Independent Samples t-Tests

The two-sample (independent groups) t-test is used to determine whether means of two populations are different from each other based on independent samples selected from each population.

Case Analysis-1

Problem

A test is conducted to compare the results of graduation students of two colleges X and Y. A random sample of 16 students of X and 18 students of Y were selected and a test is conducted on them .The marks of selected students are as follows.

Table-1: Sample Data

Student Number	Marks (College X)	Marks (College Y)
1	56	87
2	89	56

3	78	89
4	56	85
5	43	84
6	78	85
7	69	86
8	65	52
9	64	56
10	56	45
11	52	46
12	58	32
13	58	35
14	59	38
15	59	58
16	68	65
17		63
18		53

The hypotheses for the analysis are:

Null hypothesis-H_0: The students of the colleges X and Y perform equally. ($\mu_1 = \mu_2$)

Alternative Hypothesis- H_1: The average performances of the students of two colleges are not equal. ($\mu_1 \neq \mu_2$)

Input Data

The data so collected encompass two variables: first variable: - College Name (X, Y) and the second variable: Mark. The test variable is the mark and it is numeric data. The college is a group variable or coded variable. (Code = 1 for college X and code = 2 for college Y). The following table serves as the input data for the analysis.

Table-2: Input Data

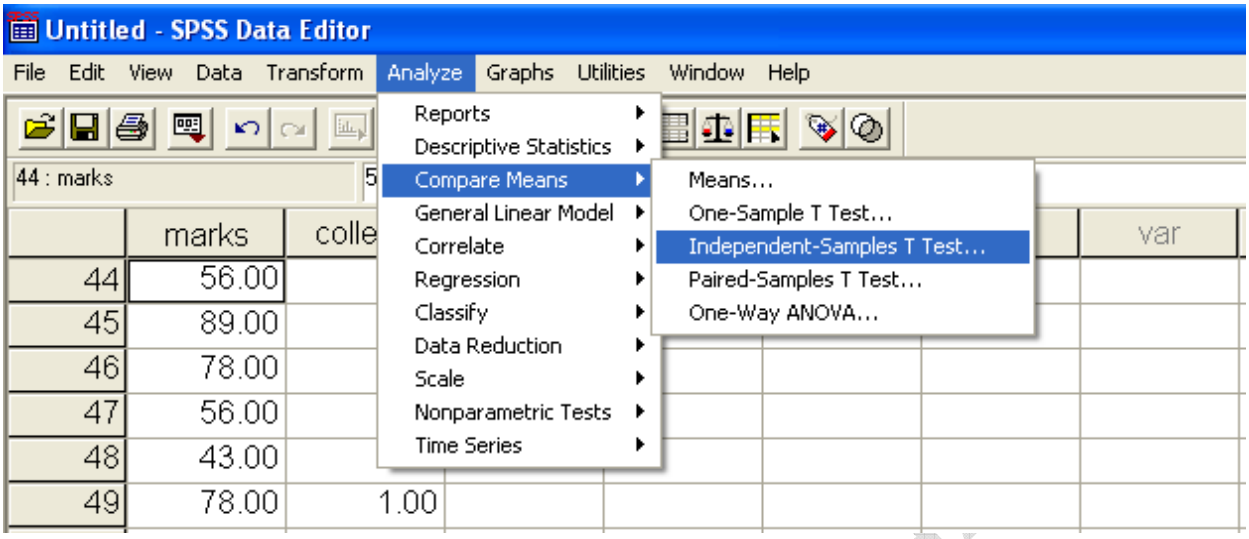
Serial Number	Marks	College (X and Y)
1	56	1
2	89	1
3	78	1
4	56	1
5	43	1
6	78	1
7	69	1
8	65	1
9	64	1
10	56	1
11	52	1

12	58	1
13	58	1
14	59	1
15	59	1
16	68	1
17	87	2
18	56	2
19	89	2
20	85	2
21	84	2
22	85	2
23	86	2
24	52	2
25	56	2
26	45	2
27	46	2
28	32	2
29	35	2
30	38	2
31	58	2
32	65	2
33	63	2
34	53	2

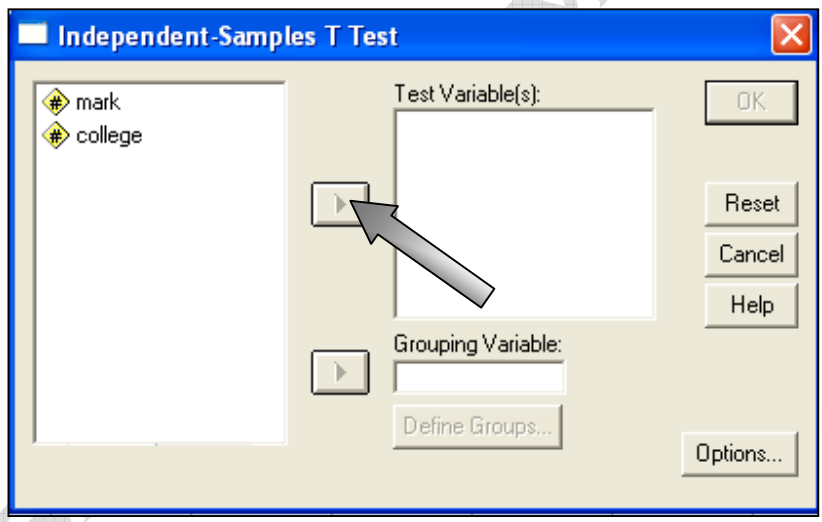
PERFORMING THE ANALYSIS WITH SPSS

FOR SPSS VERSION 11, CLICK ON ANALYZE → COMPARE MEANS INDEPENDENT-SAMPLES T TEST.

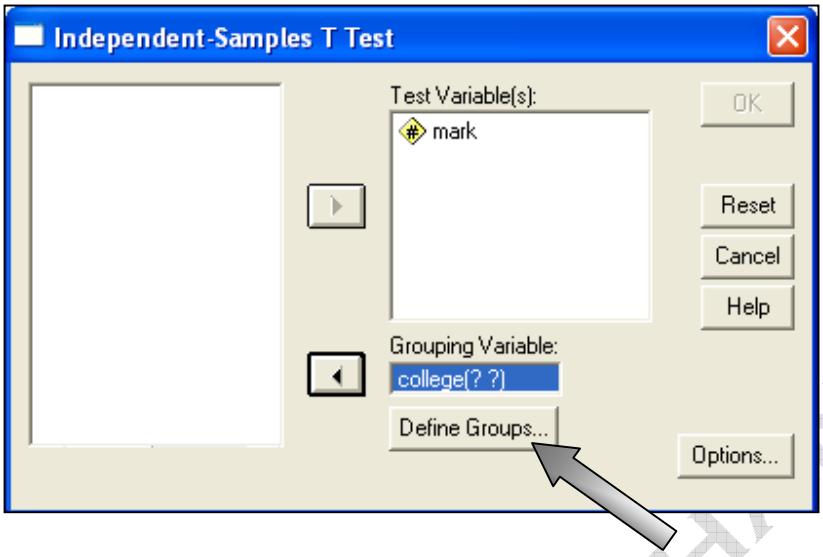
THIS WILL BRING UP THE SPSS SCREEN DIALOGUE BOX AS SHOWN BELOW.



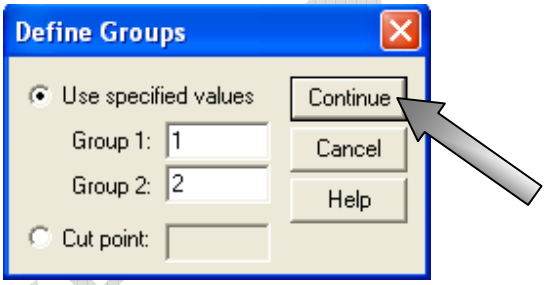
AFTER CLICKING INDEPENDENT-SAMPLES T TEST, THIS WILL BRING UP THE SPSS SCREEN DIALOGUE BOX AS SHOWN BELOW.



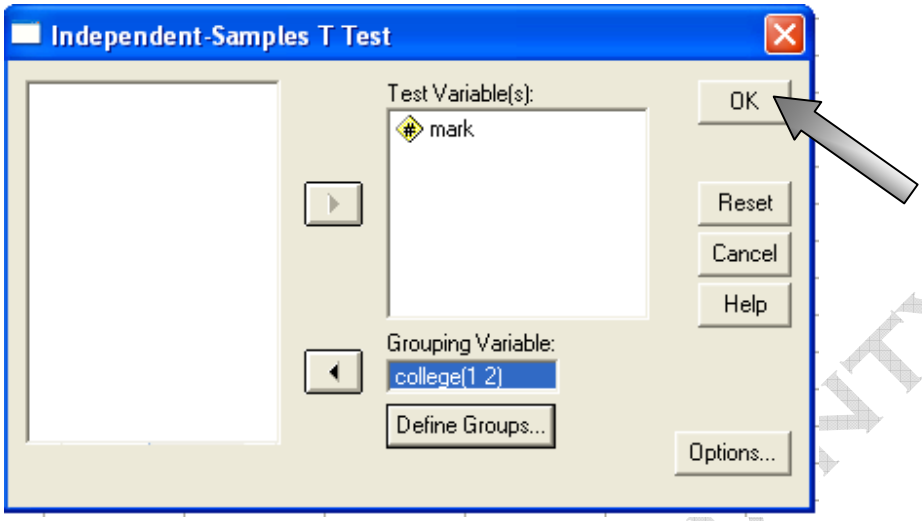
SELECT THE TEST VARIABLE AND MOVE IT TO TEST VARIABLE(S) BOX.



DEFINE THE RANGE OF VALUES OF THE GROUPING VARIABLE BY CLICKING ON **DEFINE GROUPS** JUST BELOW THE GROUPING VARIABLE BOX. THIS WILL BRING UP THE **DIALOGUE BOX** SHOWN BELOW.



GROUP VARIABLE (COLLEGE) IS DEFINED WITH THE VALUE 1 FOR COLLEGE X AND 2 FOR THE COLLEGE Y. NOW CLICK **CONTINUE** TO RETURN THE MAIN DIALOGUE BOX. THIS WILL BRING **INDEPENDENT-SAMPLES T TEST** DIALOGUE BOX AS SHOWN BELOW.



FINALLY CLICK OK.

SPSS OUTPUT

THE SPSS OUTPUTS OF THE ANALYSIS ARE GIVEN IN THE FOLLOWING TABLES.

T-TEST

Group Statistics

	COLLEGE	N	Mean	Std. Deviation	Std. Error Mean
MARKS	1.00	16	63.0000	11.37834	2.84459
	2.00	18	61.9444	19.55527	4.60922

THE TABLE TELLS THAT THE AVERAGE MARK OF 16 STUDENTS OF COLLEGE X IS 63 AND THOSE OF 18 STUDENTS OF COLLEGE Y IS 61.9444.

Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
									95% Confidence Interval of the Difference	
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
MARKS	Equal variances assumed	7.003	.013	.189	32	.851	1.0556	5.58104	-10.31265	12.42376
	Equal variances not assumed			.195	27.839	.847	1.0556	5.41633	-10.04218	12.15329

FROM THE OUTPUT, T = .195 WITH 27.839 DEGREES OF FREEDOM.

DECISION

REJECT THE NULL HYPOTHESIS IF SIG. (2-TAILED) ≤ 0.05

INTERPRETATION

THE P-VALUE IS 0.847 AND IT IS MORE THAN 0.05 (5% LEVEL OF SIGNIFICANCE), SO WE ACCEPT THE NULL HYPOTHESIS AND REJECT THE

alternative hypothesis at 5% level of significance. It is concluded that the average marks of the students of two colleges are equal.

Case Analysis-2

Problem

A study is carried out to examine whether the average number of credit cards issued per month by the bank A is more than the bank B. The data of number of credit cards issued per month by the banks A and B are recorded over ten months period are as follows.

Table-1: Sample data

Month Number	1	2	3	4	5	6	7	8	9	10
No. of credits issued by A	18	9	12	8	17	23	8	18	19	23
No. of credits issued by B	11	15	18	19	22	23	6	19	18	17

Null hypothesis-H_0: The average number of credit cards issued by two banks A and B are equal. ($\mu_1 = \mu_2$)

Alternative Hypothesis- H_1: The average number of credit cards issued by bank A is more than bank B. ($\mu_1 > \mu_2$)

SPSS Outputs

The outputs are projected in tabular form as shown below.

Table-2: Group Statistics

	Bank	N	Mean	Std. Deviation	Std. Error Mean
Credit Card	1.00	10	15.5000	5.83571	1.84541
	2.00	10	16.8000	5.07280	1.60416

Table-3: Independent Samples Test

		Levene's Test for Equality of Variances		t-test for Equality of Means						
		F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	

									Lower	Upper
Credit Card	Equal variances assumed	1.027	.324	-.532	18	.601	-1.3000	2.44518	-6.43713	3.83713
	Equal variances not assumed			-.532	17.658	.602	-1.3000	2.44518	-6.44427	3.84427

From the output, t = -0.532 with 17.658 degrees of freedom.

Decision

Reject the null hypothesis if p-value (Sig. (2-tailed)) ≤ 0.05

The p-value for one-tailed test would be $\frac{\text{Sig.}(2-\text{tailed})}{2}$

Interpretation

The p-value is $\frac{\text{Sig.}(2-\text{tailed})}{2} = \frac{0.602}{2} = 0.301$ **and it is more than 0.05 (5% level of significance), so we accept the null hypothesis and reject the alternative hypothesis at 5% level of significance. Therefore it is concluded that the average number of credit cards issued by bank A is not more than bank B.**

SPSS Command

1. Click on ANALYZE at the SPSS menu bar (in older versions of SPSS, click on STATISTICS instead of ANALYZE).
2. Click on COMPARE MEANS followed by INDEPENDENT SAMPLES T TEST.
3. Select the test variable and move it TEST VARIABLE(S) box. Then select the grouping variable and move it to GROUPING VARIABLE box. Define range and fill it with the appropriate code and click CONTINUE.
4. Select OK of the main dialogue box.

iii) Paired Sample 't' test

A paired t-test is used to compare two population means where the observations in one sample can be paired with observations in the other sample. It is generally used when:

i. **The measurements are taken from the same subject before and after a particular course of action.**

ii. The number of observations in each data set is the same, and they are related in pairs.

Case Analysis-1

Problem

The marketing head of company wants to compare the performance of two sales executives based on their average sales in last 6 weeks before and after they were trained. The average sales of salesmen A and B are given in the following table.

Table-1: Sample Data

Week	I	II	III	IV	V	VI
Average sales in units by A	24	23	16	28	20	24
Average sales in units by B	12	32	11	29	19	29

The hypotheses for the analysis are:

Null hypothesis-H_0: There is no significant difference between the average sales of A and B.

($\mu_1 = \mu_2$)

Alternative Hypothesis- H_1: There is a significant difference between the average sales of A and B. ($\mu_1 \neq \mu_2$)

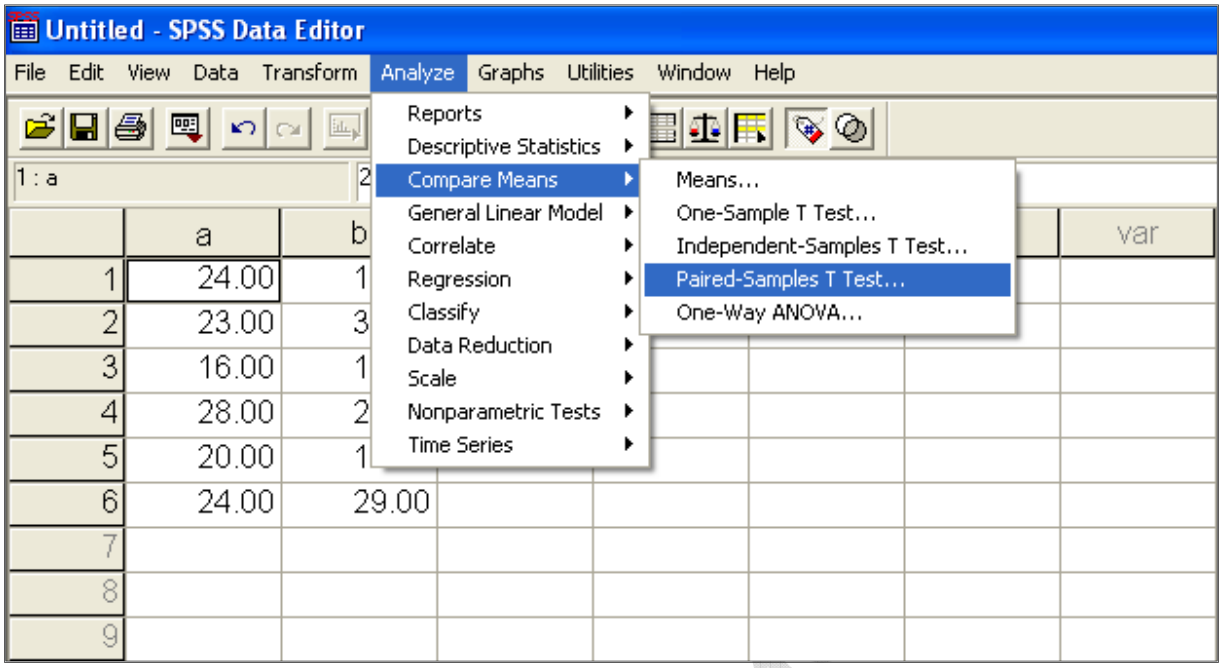
Input Data

Table-2: Input Data

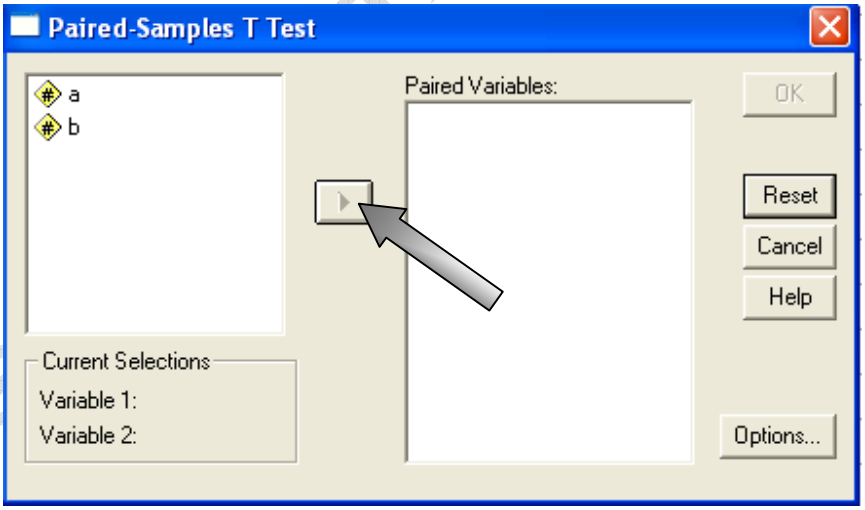
Serial Number	A	B
1	24	12
2	23	32
3	16	11
4	28	29
5	20	19
6	24	29

Performing the Analysis with SPSS

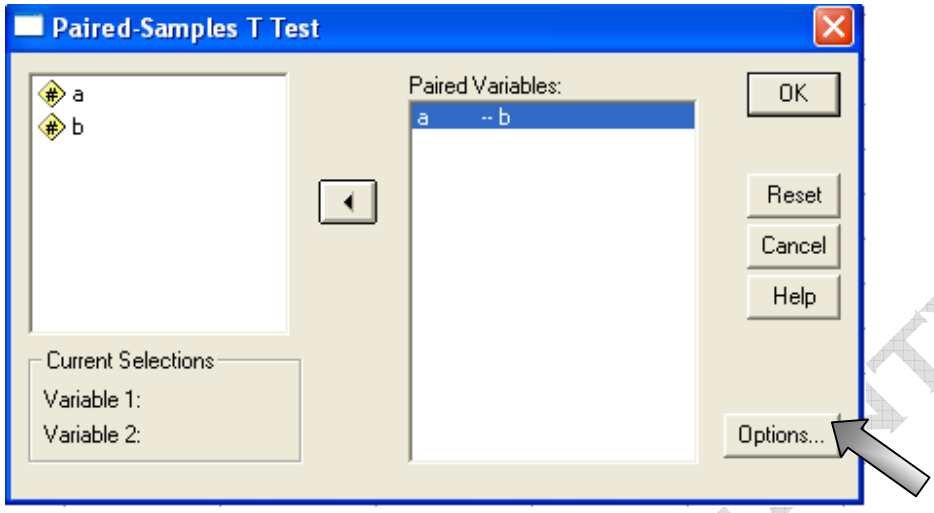
For SPSS Version 11, click on **Analyze** → **Compare Means** **Paired-Samples T Test** .This will bring up the SPSS screen dialogue box as shown below



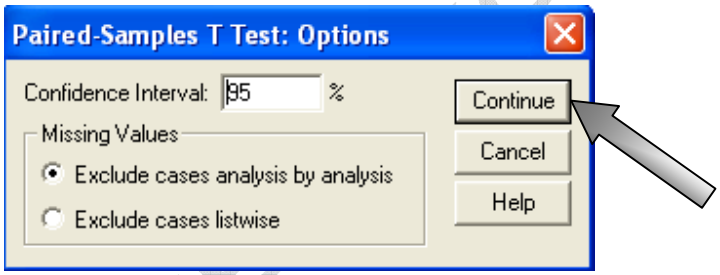
AFTER CLICKING PAIRED-SAMPLES T TEST, THIS WILL BRING UP THE FOLLOWING SPSS SCREEN DIALOGUE BOX



SELECT THE VARIABLES AND MOVE THEM TO PAIRED VARIABLES BOX.



CLICK **OPTION** AND SELECT CONFIDENCE INTERVAL 95% (5% LEVEL OF SIGNIFICANCE) AND THEN **CONTINUE**.



THIS WILL BRING THE **PAIRED-SAMPLES T TEST** DIALOGUE BOX. FINALLY CLICK OK.

SPSS OUTPUT

THE SPSS OUTPUTS OF THE ANALYSIS ARE DEPICTED IN TABLE-3 AND TABLE-5

T-TEST

TABLE-3: PAIRED SAMPLES STATISTICS

		MEAN	N	STD. DEVIATION	STD. ERROR MEAN
PAIR 1	A	22.5000	6	4.08656	1.66833
	B	22.0000	6	9.25203	3.77712

THE AVERAGE SALES BY THE SALESMAN A IS 22.5 UNITS AND 22 UNITS BY B.

TABLE-4: PAIRED SAMPLES CORRELATIONS

		N	CORRELATION	SIG.
PAIR 1	A & B	6	.624	.185

The correlation between A and B is 0.624

Table-5: Paired Samples Test

		Paired Differences					T	df	Sig. (2-tailed)
		Mean	Std. Deviation	Std. Error Mean	95% Confidence Interval of the Difference				
					Lower	Upper			
Pair 1	A - B	.5000	7.42294	3.03040	-7.2899	8.2899	.165	5	.875

From the output, t = 0.165 with 5 degrees of freedom

Decision

Reject the null hypothesis if p-value (Sig. value) ≤ 0.05

Interpretation

The p-value is 0.875 and it is more than 0.05 (**5% level of significance**), so we accept the null hypothesis and reject the alternative hypothesis at 5% level of significance. It is concluded that the average sales by two salesmen are equal.

Case Analysis-2

Problem

The pharmaceutical company is in process of introducing a new drug and it is claimed that the drug can effectively be used in reducing the blood pressure. To test the claim, a sample of 10 patients is selected and their blood pressure before and after using the drug are recorded as given below.

Table-1: Sample Data

Patient Number	1	2	3	4	5	6	7	8	9	10
B.P before using the drug	121	115	112	123	108	112	119	125	129	116
B.P after using the	116	118	116	120	112	115	109	110	109	119

DRUG										

THE HYPOTHESES FOR THE ANALYSIS ARE:

NULL HYPOTHESIS-H_0: THERE IS NO DIFFERENCE IN THE BLOOD PRESSURE BEFORE AND AFTER USING THE DRUG. ($\mu_1 = \mu_2$)

ALTERNATIVE HYPOTHESIS- H_1: THE BLOOD PRESSURE AFTER DRUG IS LESS THAN THE BLOOD PRESSURE AFTER THE DRUG. ($\mu_1 < \mu_2$) **(ONE TAILED TEST)**

TABLE-2: PAIRED SAMPLES STATISTICS

		MEAN	N	STD. DEVIATION	STD. ERROR MEAN
PAIR 1	BEFORE	118.0000	10	6.58281	2.08167
	AFTER	114.4000	10	4.14193	1.30979

TABLE-3: PAIRED SAMPLES CORRELATIONS

		N	CORRELATION	SIG.
PAIR 1	BEFORE & AFTER	10	-.293	.411

TABLE-4: PAIRED SAMPLES TEST

		PAIRED DIFFERENCES					T	DF	SIG. (2-TAILED)
		MEAN	STD. DEVIATION	STD. ERROR MEAN	95% CONFIDENCE INTERVAL OF THE DIFFERENCE				
					LOWER	UPPER			
PAIR 1	BEFORE AFTER	3.6000	8.74579	2.76566	-2.6564	9.8564	1.302	9	.225

FROM THE OUTPUT, T = 1.302 WITH 9 DEGREES OF FREEDOM.

DECISION

REJECT THE NULL HYPOTHESIS IF P-VALUE (SIG. (2-TAILED)) ≤ 0.05

THE P-VALUE FOR ONE-TAILED TEST WOULD BE $\frac{\text{Sig.}(2-\text{tailed})}{2}$

INTERPRETATION

THE P-VALUE IS $\frac{\text{Sig.}(2-\text{tailed})}{2} = \frac{0.225}{2} = 0.1125$ AND IT IS LESS THAN 0.05 (5% LEVEL OF SIGNIFICANCE), SO WE REJECT THE NULL HYPOTHESIS AND ACCEPT THE

alternative hypothesis at 5% level of significance. Therefore it can be concluded that the drug can be used reduce the blood pressure.

SPSS Command

1. Click on ANALYZE at the SPSS menu bar (in older versions of SPSS, click on STATISTICS instead of ANALYZE).
2. Click on COMPARE MEANS followed by PAIRED SAMPLES T TEST.
3. Select the variables and move them to PAIRED VARIABLES box.
4. Click OPTION and select confidence interval 95% (5% level of significance) and then CONTINUE.

Select OK of the main dialogue box.